NONSTATIONARY HEAT TRANSFER

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## 1. Introduction

The experimental and theoretical heat-transfer intensity relationships now widely employed in heat-transfer calculations for technological processes and heat-exchange equipment were obtained for stationary heat-transfer conditions. Under actual conditions, parameters of importance to the heat-transfer process often vary both during the process itself and in the course of transients. In much modern equipment, the time rates of change of the thermal parameters can be high.

This raises the question of whether these relationships remain applicable under such conditions.

The literature contains many theoretical and substantially fewer experimental studies of nonstationary heat exchange, which is important for several new fields of science and technology.

To compute the heat-transfer intensity, we use the expression

$$q_{\mathrm{w}} = \alpha (T_{\mathrm{w}} - T_{\mathrm{fl}}). \tag{1}$$

In actual processes, such parameters as $T_w$, $T_{fl}$, the flow velocity, and the properties of the fluid that determine the heat-transfer intensity can vary with time. For identical values of $T_w$ and $T_{fl}$, the heat fluxes will in principle differ for stationary and nonstationary conditions owing to the difference in the heat-transfer coefficients. To determine the ratio of these coefficients, we must, by theoretical or experimental means, find the way in which this ratio depends on the heat-transfer parameters relevant for stationary conditions and on the new parameters characteristic of the nonstationary process. For these latter parameters, it makes sense to take the rate of change of the time-dependent heat-transfer parameters, such as $\partial T_w/\partial \tau$, $\partial T_{fl}/\partial \tau$, $\partial w/\partial \tau$, where w is the flow velocity.

The degree of deviation from a stationary process depends both on the rate of change of the wall temperature and on the properties of the medium. We can imagine a case in which high heat capacity and low thermal conductivity of the medium can produce significant deviations in the heat-transfer rate from the corresponding stationary values at relatively low values of the heat-transfer rate. The reason is that the amount by which the process differs from the stationary mode is in general not determined by the absolute time rate of change in the temperature of the surface, but by an entire complex of parameters that includes the properties of the medium moving with respect to the body.

Thus the important problems in nonstationary heat transfer include not only the case of stepwise change in parameters, but also the case of slower variation of, for example, heating and cooling of bodies, since here we also have a substantial departure of the stationary processes. These cases are also important since it is just these that we face in practice.

This sort of nonstationary heat-transfer process naturally does not exclude cases in which despite a variation in, for example, the surface temperature, the temperature profiles in the boundary layer can be adjusted in accordance with the changing conditions at the wall, and ordinary methods can be used in heat-transfer calculations, with the instantaneous parameter values being employed. In such case, we have a quasistationary heat-transfer regime.

This situation, in which we have joint nonstationary heat conduction in the boundary layer and wall, differs from the situation in which the rate of heat propagation is finite [1, 2]. As was shown in [1, 2], for

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most cases encountered, the expressions for the heat flux in ordinary form yield sufficiently accurate results. If we neglect the partial derivative of the heat flux with respect to time, the resulting error in determination of the time at which a certain temperature value appears in a given section amounts to $10^{-9}$–$10^{-13}$ sec, which is hardly noticeable, given the present-day state of measurement technique.

These conclusions obtained by examining the mechanism of the process can be confirmed by theoretical analysis.

The authors restrict their discussion to studies involving the conditions of ordinary thermal-equipment engineering, and do not consider nonstationary heat exchange, which is complicated by other processes (chemical transformations, dissociations, etc.).

## 2. Theoretical Studies

We shall take a brief look at the results of the theoretical analysis of the nonstationary heat-transfer process (on the basis of published data).

The nature of the nonstationary heat-transfer process is such that the boundary conditions are not known in advance, but are found during solution of the problem. This requires combined examination of the heat-conduction equations for the solid body and the energy equations for the fluid flow, i.e., the study of nonstationary heat transfer involves a coordinated problem [4-7].

As several authors have noted [8, 44], at the present time it is a very complicated matter to solve such a problem.

The problem was first raised in [5]. The authors considered nonstationary heat exchange between a section of rectangular type and a laminar fluid flow. The process is nonstationary owing to cooling of the pipe section. Allowance was made for the temperature distribution over the thickness of the pipe. At the initial time, the wall temperature (constant over the thickness) differs from the flow temperature. Many of the ordinary assumptions were used in solving the problem.

Simultaneous solution of the wall heat-conduction equation and the flow energy equation, allowance for the boundary condition of the system as a whole, yielded the following relationship for the specific heat flux through the heat-exchange surface at high values of $\xi$ [5]:

$$q \cong \lambda_{fl} t_0 \sqrt{\frac{\mathrm{Pe}}{dl}} \exp\left[-\lambda_{fl} \sqrt{\frac{\mathrm{Pe}}{dl}} \frac{\tau}{(c\rho)_w \delta}\right], \tag{2}$$

where $t_0$ is the initial temperature of the section of the wall of the tube; d is the equivalent diameter of the tube; $l$ is the dimension of the section of the tube; $\delta$ is the thickness of the considered section of the tube; $\xi = d\mathrm{PeFo}_{fl}/l$.

It follows from the relationship obtained that the rate of nonstationary heat transfer is influenced both by usual heat-transfer parameters for stationary cases (such as Re, Pr, and the characteristic length d), and by the properties of the wall material $(c\rho)_w$, the wall thickness $\delta$, and the initial temperature $t_0$. The power of $\delta$ and $c\rho$ in the exponent will differ, for the same values of Bi, from the power for the quasistationary exponential relationship (with $\alpha$ = const). This is obvious from the physical viewpoint since, depending on the thickness and material of the wall, the temperature distributions in the wall, including that at the heat-exchange surface, will differ for identical times and identical hydrodynamic conditions. As a consequence, the temperature gradients at the surface and the heat fluxes will also differ. The values of the heat fluxes and of $\alpha$ will differ from the corresponding values for quasistationary conditions. Here we also note that the basic differences between nonstationary and stationary heat transfer were first formulated in [3-5].

The question of the difference between heat-transfer coefficients under nonstationary and stationary conditions was raised by Kudryavtsev in 1948 [3]. In [4], the features of nonstationary exchange were confirmed experimentally.

The first solutions of nonstationary problems were found in 1959 and 1960 [8-10]. Nonstationary heat transfer in a pipe with stabilized flow was also considered for the initial thermal section of the pipe [9, 10]. In contrast to [5], the coordinated problem was not considered; it was assumed that at a certain time the wall temperature or the heat flux changed abruptly to a new constant value. As a result, the heat flux density q at the wall was attained as a function of the pipe coordinate x/d and Fo for a temperature jump. As Fo increases, $q_w$ decreases, approaching a constant value that depends on x/d. Calculations show [8] that

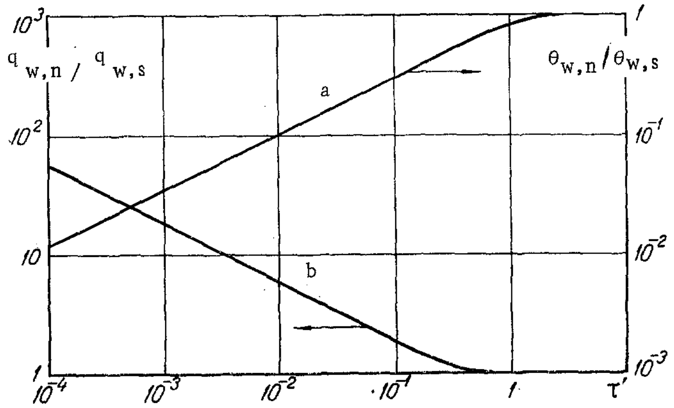


Fig. 1. Ratios of heat fluxes and surface temperatures at frontal point of sphere under nonstationary and stationary conditions as functions of the parameter $\tau' = Pr^{-1/4}U_\infty t/R$ [18]: a) step change in heat flux at surface; b) step change in temperature at surface.

for certain values of x/d and Pe, a new stationary state sets in after 18 sec for transformer oil, 8 sec for water, and 0.05 sec for air.

For a stepwise doubling of the heat flux over the initial flux for real values of the flow parameters in a pipe, the time required for the nonstationary heat-exchange process to settle to a new state will be of the order of 90 sec.

When the pressure gradient varies periodically in time (pulsating flow), we also will have a process of nonstationary heat transfer; the intensity of the process will differ from that of stationary heat transfer, depending on the pulsation frequency [8]. As the frequency drops, the process becomes quasistationary. The Prandtl number has the following influence: if for a specific frequency with the other parameters being constant, the heat-transfer process is quasistationary for $Pr \ll 1$, then for sufficiently high values of Pr, the nonstationary heat-transfer rate will substantially exceed the stationary heat flux. The difference in the rates increases roughly in proportion to the amplitude of the oscillations.

Making the usual assumptions, the author of [11, 12] reduced the problem to solution of the heat-conduction equation, and obtained relationships for $Nu_n$ for many specific cases.

Analysis of the nonstationary heat-transfer process for a linear time variation in wall temperature [12] indicates that the value of the nonstationary criterion $Nu_n$ will be a multiple of the value of $Nu_s$ (for stationary conditions) during the initial time period.

When the wall temperature of a channel varies exponentially, $Nu_n$ will increase without limit in the course of time.

Flow of an incompressible fluid in a pipe has been considered in [13] for temperature that varies in time and along the axis of the channel. In addition to certain general assumption, it was assumed that the ratio of the difference of the tube radii to the inside radius is small, that the fluid velocity is constant in time and over a cross section, and that the heat capacity of unit volume of fluid can be neglected as compared with the heat capacity of unit volume of the wall. The results indicate that it is necessary to allow for the way in which the nonstationary heat conduction of the wall affects the heat-transfer process, i.e., it is necessary to allow for the influence of the thickness (even where the relative thickness is not large) and the thermal characteristics of the wall material.

The nonstationary thermal laminar boundary layer at a plate has been considered in [14-17, 54]. The process is nonstationary owing to the abrupt initial motion of the isothermal plate with respect to the fluid [14, 15] and the step variation in heat release [16, 17, 54]. It was shown that there are nonstationary, transient, and quasistationary regimes; the times at which these processes set in depend, in particular, on Pr. Expressions were obtained for Nu in each of these regimes.

It was also shown that the heat capacity of the plate exerts a significant influence on the change in temperature even for a thickness of 0.1 mm (foil was used). Certain other cases of nonstationary heat transfer for a plate have been considered in [46-51].

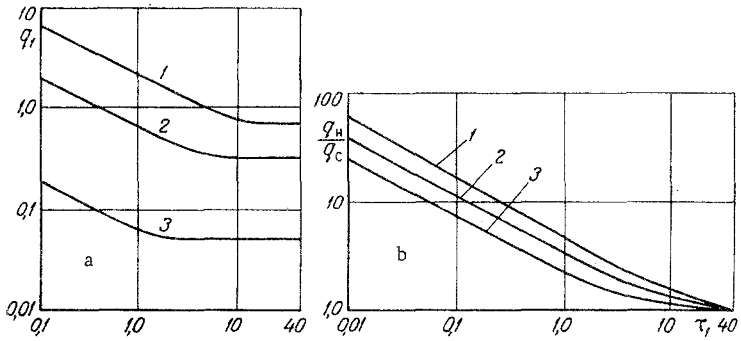


Fig. 2. Relative heat flux at plate surface as function of parameter $\tau_1 = U_\infty \tau/x$ under variation in surface temperature [54]: a) results for step variation in surface temperature ($q_1 = q/K\sqrt{Re}(T_W - T_\infty)/x$); b) results for linear increase in surface temperature; 1) Pr = 10; 2) 1.0; 3) 0.01.

The authors of [18] were able to make an explicit demonstration of the difference in heat-transfer rates for corresponding stationary and nonstationary conditions. They considered the leading edge of a plate and the stagnation point of an axisymmetric body for a stepwise change (from one constant value to another) in the temperature and heat flux at the surface. The oncoming flow was assumed to be hydrodynamically stationary and the fluid to be incompressible with constant properties; the energy dissipation in the boundary layer was neglected. The problem was solved separately for short and long time segments. The authors' solution for the stationary cases agree with the familiar Sibulkin and Mekhcin solutions. It follows from the results (Fig. 1) that for a step change in surface temperature, the ratio of the nonstationary flux to the stationary flux can reach several orders of magnitude during the initial time period, depending on Pr, the velocity $U_\infty$ of the oncoming flow, and the radius of curvature R of the front of the body. In particular, as Pr increases, the ratio becomes greater. The ratio of the heat fluxes approaches unity in the course of time. The time required for the stationary state to become established is inversely proportional to the velocity of the undisturbed flow and directly proportional to the fourth root of Pr.

Nonstationary heat transfer at the stagnation point of a blunt body has been dealt with in several other studies as well. Arbitrary change in the velocity of the oncoming flow was considered [52], as was nonstationary heat transfer in compressible boundary layers [53].

The ratio of the nonstationary heat flux to the stationary flux can also differ from unity in the case of forced convection for a laminar flow past a flat plate, where the temperature of the surface changes abruptly (Fig. 2a) and linearly (Fig. 2b).

We can draw certain conclusions from the theoretical results.

1. Theoretical analysis of nonstationary heat transfer confirms the previous statements based on examination of the mechanism for nonstationary heat transfer.

2. The problem of nonstationary heat transfer is in principle a "coordinated" problem. This does not exclude other approaches.

3. If the wall heat-conduction equation is replaced by boundary conditions such as the frequently utilized stepwise variation of the heat flux or temperature at the surface, the problem becomes simpler, but the region of applicability of the results contracts. The Duhamel principle can be used to generalize the results for stepwise parameter variation to monotonic variation, which corresponds better to the actual processes.

Let us now look at the methods and results of experimental investigations into nonstationary heat transfer.

## 3. Methods for Experimental Determination of the Boundary Condition of Nonstationary Heat Transfer

The following methods can be used to set up the conditions for nonstationary heat transfer: 1) stepwise variation in the heat flux or the temperature at the surface (this method is difficult to realize [16,19]); 2) stepwise variation in the heat released within the body [19-26], realized by changing the applied electric load; 3) heating or cooling of the body in the fluid flow [4, 20-22, 27-32]; 4) variation of the flow temperature [26, 33]; 5) variation of hydrodynamic parameters (such as the velocity [34] or fluid flow rate [20-22, 26, 32, 35]); 6) periodic variation of parameters (pulsating flows).

1. Determination of Heat-Transfer Coefficient at Low Values of Bi (exponential method [3]). In this case, the amount of heat applied to the body during the interval $d\tau$ equals the heat capacity of the body multiplied by the change dt in the body temperature. In this case, the temperature is the same for all points in the body, so that

$$Qd\tau = Gc_p dt = \alpha(\tau)(t_{fl} - t) F d\tau. \tag{3}$$

If we let $t_{fl} - t = \vartheta$, the radius of the sphere equal R, and the plate thicknesses be $\delta$, we obtain the following simple formulas for the heat-transfer coefficient:

for a sphere,

$$\alpha(\tau) = -\frac{c\rho R}{3}\frac{d\ln\vartheta}{d\tau}, \tag{4'}$$

for a thin-walled cylinder and a plate cutout,

$$\alpha(\tau) = -c\rho\delta\frac{d\ln\vartheta}{d\tau}. \tag{4''}$$

Here $\alpha$ can be an arbitrary time function. For stationary heat transfer, the derivatives in these formulas will be constant. For a constant heat-transfer coefficient, we obtain the following relationship from (4) as a special case: $\alpha = -(c\rho R/3\tau)\ln(\vartheta/\vartheta_0)$; this is used in [29, 36, 37], for example. In this case, $\ln\vartheta$ is a linear function of $\tau$.

This method has been used in [30, 34, 38] for a hollow cylinder, in [28] for a hollow sphere, and in [39, 40] for spherical particles.

The assumption that Bi is small restricts the applicability of the given method and, moreover, often leads to various errors in the results, since under actual conditions there will be temperature drops over the thickness, and it will be difficult to determine in advance (without knowing the heat-transfer coefficient) the dimension that must be assumed to be small. Certain errors inevitably occur when the derivative in (4) is replaced by the ratio of the finite increments.

2. Determination of Heat-Transfer Coefficient for Large Bi. A) Method of Successive Intervals [4]. Replacing the monotonic dependence of the heat flux at the plate surface on the time by a step curve, the authors have given solutions of problems for successive time intervals. The values found at the end of a given interval form the initial conditions for the solution in the next interval. In the solution for each interval it was assumed that the region Fo $\geq 0.5$ is considered.

Without introducing any great error into the temperature determination, we can neglect the infinite sum of the terms of the series as compared with the first terms of the solution. The solution for the temperature at the n-th time interval yields an expression for the heat flux,

$$q_n = \frac{[t(x,\tau) - t_0]\dfrac{\lambda}{\delta} - \sum\limits_{i=1}^{i=n-1} q_i \mathrm{Fo}_i}{\mathrm{Fo}_n - \dfrac{1}{6} + \dfrac{x^2}{2\delta^2}}. \tag{5}$$

This relationship can be generalized to the case of variable thermal properties [4].

When (5) is employed, there may be certain errors in the results. There will inevitably be an error in determination of the heat flux during the initial time period (where it is greatest) owing to the fact that while the variations in temperature and heat flux at the surface are significant, the change in temperature far from the surface will be relatively small. The determination of heat-transfer parameters at the surface in terms of small changes in temperature far from the surface can introduce substantial error. By the very essence of the method, this error will be introduced into the succeeding heat-flux values. Errors can also be associated with the replacement of the actual relationship by a step curve (particularly when there are large changes in the heat flux in unit time). The situation is made still worse by the fact that the time interval corresponding to one step cannot be made smaller than the value found from the condition Fo $\geq 0.5$.

The errors may be reduced when there is a decrease in the distance between the location of the thermocouple junction and the surface. This can be done, in particular, by reducing the thickness of the calorimeter element when the junction is located at the rear surface. Under these conditions, comparison of the successive-interval method and the exponential method has shown [41] that the two methods give heat-flux measurement results that are in good agreement.

To determine the heat-transfer coefficient by the successive-interval method, we must know the temperature of the fluid flow, and use (5) to determine the temperature of the specimen surface. The latter determination is subject to the same errors as are the heat-flux data.

B) The Mean-Temperature Method [4]. In this method, we make use of one aspect of the heating (cooling) of a body: there always is a certain isothermal surface in the body, whose temperature at each instant equals the mean temperature of the body. When Fo ≥ 0.5, the coordinate of this plane for a plate, under boundary conditions of the second kind, is determined from the condition

$$\delta^2 - 3x_*^2/G\delta^2 = 0, \tag{6}$$

while the solution of the heat-conduction equation reduces to

$$t(x_*, \tau) - t_0 = \frac{q}{\lambda}\frac{d\tau}{\delta}. \tag{7}$$

Comparing (4) and (7), we see that the temperature in (7) is the mean temperature of the body.

It was shown in [4] that (6) is also valid under other boundary conditions. In this method, the heat flux is determined by means of (7) from the temperature measured at the point $x_*$. The method is subject to all the drawbacks of the successive-interval method.

C) The Surface-Point Method [42]. A method for determining nonstationary boundary conditions can be obtained by solving the heat-conduction equation for boundary conditions varying arbitrarily in time. In [42], the problem was solved for plates under the assumption that the arbitrary time relationships of the temperatures at both plate surfaces were specified. For the zero initial condition, the solution of such a problem has the following form [42]:

$$t(x, \tau) = \varphi_1(\tau) + \frac{x}{\delta}[\varphi_2(\tau) - \varphi_1(\tau)] + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left[-\left(\frac{\pi n}{\delta}\right)^2 a\tau\right]$$
$$\times \sin\frac{\pi n}{\delta}\int_0^{\tau}\exp\left[\left(\frac{\pi n}{\delta}\right)^2 a\tau^*\right][(-1)^n\varphi_2(\tau^*) - \varphi_1(\tau^*)]\,d\tau^*. \tag{8}$$

The heat flux at the surface can be found if we take the derivative of the temperature (8) with respect to x, set x to zero, and substitute the derivative into the heat-conduction law (9):

$$q = -\lambda\left.\frac{\partial t}{\partial x}\right|_{x=0}. \tag{9}$$

We must emphasize that the method is free of restrictions on the values of Bi, Fo that were employed in the preceding methods. There are difficulties associated with measurement of the temperature of the surface at which heat transfer takes place.

D) Determination of the Heat Flux at the Surface of a Semiinfinite Body. When the wall can be treated as a semiinfinite boundary, we need only measure the temperature at the surface to determine q [43]. If we find the derivative of the temperature with respect to the coordinate at the surface and substitute it into (9), we obtain an expression for the heat flux at the surface [43],

$$q_{(\tau)} = \frac{\lambda}{\sqrt{\pi a}}\int_0^{\tau}\frac{d\varphi(\tau^*)}{d\tau}\,\frac{d\tau^*}{(\tau-\tau^*)^{\frac{1}{2}}}. \tag{10}$$

E) Determination of the Heat-Transfer Coefficient for Motion of a Gas in a Pipe [20, 44, 45]. The heat-transfer coefficient for nonstationary coefficients is determined, as for stationary conditions, by the expression

$$\alpha = \frac{q_{\mathrm{w}}}{t_{\mathrm{fl}} - t_{\mathrm{fl}}}. \tag{11}$$

In [44], the one-dimensional energy equation was solved for the mean-calorimeter temperature of the flow to determine the fluid temperature at any section for any instant $t_{fl}(x, \tau)$; the following were assumed to be known: the initial temperature and flow rate, and the relationship between the heat flux $q_W$ and the inside surface of the pipe and x and $\tau$.

The mean-calorimeter temperature, found by a numerical method, was taken to equal the unknown fluid temperature $t_{fl}$.

To determine the heat flux and temperature at the inside surface of the pipe, the heat-conduction equation for the tube wall was solved. Here it was assumed that there is no flow of heat along the pipe through the wall, that the heat sources are distributed uniformly over the wall thickness, and that the heat release is determined by the specified value of $q_V$. The solution was obtained by computer with the aid of a finite-difference method. With the relationships found, the measured values of outside tube-surface temperature and heat release $q_V$ could be used to find the heat flux and the temperature at the inside surface. In contrast to the initial formulation of the problem, at this stage it was assumed that $q_W$ and $t_W$ are time-dependent, but independent of the coordinates. Thus a fairly complex computational method was used to find the quantities that determine the heat-transfer coefficient in accordance with (11). One advantage of such a formulation is that it very closely approximates one actual case of nonstationary heat transfer.

Depending on the actual conditions, several other methods can be developed to determine the heat-transfer coefficients that resemble those considered in paragraphs C), D), and E).

## 4. Results of Experimental Investigation of Nonstationary Heat Transfer

There have been several studies of heat transfer for a body for which Bi < 0.1-0.01 (depending on the accuracy required) in a stream of air.

Wind-tunnel studies have been made for Re values of between 650 and $1.3 \cdot 10^5$ [36] and $2.4 \cdot 10^4$ to $2.4 \cdot 10^5$ [37, 29]. The experimental objects were 6.32 and 3.93 mm diameter steel balls [36], and balls made from various metals, and measuring 30-80 mm in diameter [37, 29]. The balls were first heated in electric furnaces and then placed in a stream of air. The initial temperature drop reached 30°C [36] and 80°C [37, 29]. The cooling process was monitored by thermocouples and an oscilloscope. The resulting temperature curves were used to determine the heat-transfer coefficient from the first of the methods discussed. The experimental results indicate that under such conditions the heat-transfer coefficient is independent of the cooling rate, the size of the ball, or the material of the ball, and is constant for a given value of Re. In our opinion, the absence of unsteady effects during the experiments is accounted for by the low rates of change of temperature with time. At the initial stage of the process, the rates did not exceed 0.5-2.0°C/sec; particularly for heat transfer to air, such values make it possible for the boundary layer to change in accordance with the temperature of the surface. The changes in ball size and in $c\rho$ that occurred in the experiments has little influence on this value.

The experimental bodies were also selected on the basis of a small Bi number in [28, 34, 30, 40].

In [28, 30], unsteady heat transfer was investigated for hollow spheres and cylinders in an air stream; a wind tunnel was used. The value of Re varied from $10^3$ to $7 \cdot 10^3$. In [28], the experimental objects were thin-walled hollow copper ($\delta$ = 0.2 mm) and aluminum ($\delta$ = 0.36 mm) spheres, 39 mm in diameter, and a hollow duralumin cylinder, 36 mm in diameter, with 0.25 mm thick wall. The cylinder was heated in a thermostat to 180°C, and introduced into the flow. The spheres were heated in a furnace, and placed in a wind tunnel. After the specimens had been heated to the required temperatures, the furnace was removed, and the cooling process commenced. The temperatures were recorded by means of thermocouples using 0.1 mm diameter electrodes in conjunction with a type GZS-47 galvanometer. The experimental data was processed in terms of the ratio $Nu_n/Nu_{st}$; an exponential method based on Re and Fo was employed. Special experiments were used to determine $Nu_{st}$ (for steady-state conditions); the results are in agreement with the published relationships. The experimental results can be described by an expression of the form

$$\frac{Nu_n}{Nu_{st}} = 1 + \frac{C}{Fo^m Re^n},$$

where the constants depend on the shape of the body and the range of Fo and Re values.

It follows from [28, 30] that the heat-transfer coefficient is larger under unsteady conditions than under steady. The ratio diminishes with time and with increasing flow velocity. It also depends on the shape of the body. Most of the change in the coefficient occurs during the first 2-4 sec, with the coefficient becoming almost constant after 10-15 sec.

In [34], an unsteady heating process in a hollow 8.4 and 2.6 mm diameter cylinder was induced by abruptly placing it in a transverse jet of air heated to 70-80°C. The value of Re ranged from 400 to 3700. The cylinder, formed by winding 0.1 mm diameter wire over paper, serves simultaneously as a resistance thermometer for determining the average temperature over the circumference. The measurements established that during the first 5-10 sec, the heat-transfer coefficient rises from nearly zero to the steady-state value.

Comparing the air-flow parameters employed in [36, 37, 29, 28, 34, 30], we see that the parameter ranges overlap or are close. All of the experimental specimens satisfy the condition $Bi < 0.1$. The initial temperature drop amounted to several dozen degrees. The body was then either rapidly placed in a flow, or else a gate obstructing the flow was removed. Recording of the specimen temperatures began during the first few seconds. The same method was used in all the studies to determine the heat-transfer coefficient (it was required that Bi be small).

The results of the experiments differ. In [36, 37, 29], the coefficient remained constant for the first few seconds, equal to the steady value; in [28, 30], it exceeded the steady value during the first few seconds, then approached a constant value; in [34] finally, during the initial period of the process the coefficient was below the steady value, which it then approached.

These results, apparently contradictory, can be accounted for in the following way.

In [28, 30], the bodies employed were thin-walled, and were initially heated to 180°C. Thus during the initial cooling period, it was possible to obtain greater rates of change of temperature with time than for the solid spheres employed in [29], with initial heating to 80°C. Moreover, the authors of [28, 30] concentrated on recording the temperature during the first 2-4 sec of the process. The rates of temperature variation recorded in [28, 36] reached 5-6°C/sec. (In [37, 29], they did not exceed 0.5-2°C/sec.) For such rates of change in air, unsteady-state effects can occur [22], but the $Nu/Nu_{st}$ ratio of 2-2.5 obtained in [28, 30] is obviously too high.

In [34], the method employed to create unsteady conditions differed from that used in the other studies mentioned. There were also differences in the direction of the heat flux, the arrangement of the experimental body, and the method used to measure temperature. These factors may account for the result discussed.

The notion that there is a fundamental difference between unsteady and steady heat exchange was first formulated in [4], where it was confirmed experimentally.

The investigation was carried out in a stream of water. A thermostat was used, in conjunction with a setup consisting of a thermostat, a pump, and a pipe, all forming a closed loop. The value of Re varied from 200 to 5000. A pipe of rectangular cross section was used. The investigated body was so installed at the pipe wall that its uninsulated end was integral with the inside surface of the pipe. The specimens were solid cylinders, 10-100 mm long (15 mm in diameter), made from various metals. Thermocouples (Ch – C type, 0.2 mm in diameter) were bounded at various distances from the end of the specimen at which the heat-exchange process was taking place. The specimens were first cooled in a thermostat to 0°C. The temperature of the water in the thermostat employed to investigate the heating process was 74°C, while the water stream in the pipe was at 46°C. A type POB-12 oscilloscope was used to record the thermocouple readings. The authors employed their own mean-temperature and successive-interval methods to process the temperature curves so as to find the boundary conditions at the heat-exchange surface.

The results obtained amount to the following. The heat flux varying with time during heating was greater at any given instant the thicker the specimen (cut from a plate). The $q - \tau$ relationship is also determined by the parameter $c\rho$ of the material. These facts are obvious, since as, for example, the thickness increases, there is a slow rise in surface temperature with time and, consequently, the heat flux is greater at each instant for $t_f = \text{const}$. The authors showed that by replacing the time by the variable $\tau/Rc\rho$, where R is the thickness of the specimen, the heat-flux curves can be reduced to a single curve.

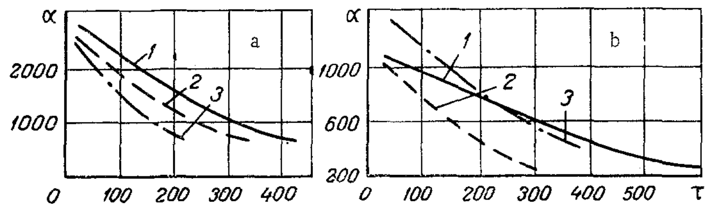


Fig. 3. Coefficient $\alpha$ (kcal/m$^2 \cdot$h$\cdot$°C) as function of $\tau$ (sec) [4]: a) 1) R = 100 mm (copper); 2) R = 75 mm (silver); 3) R = 50 mm (copper); b) 1) R = 100mm (silver); 2) R = 50 mm (silver); 3) R = 50 mm (cobalt).

One essentially new fact appeared: the heat-transfer coefficient, obtained as the ratio of the heat flux at a given time to the difference in the temperatures of the surface and the fluid is a variable that depends on the time, the thickness, and the parameter $\rho c$ of the specimen (Fig. 3). For a 75 mm thick silver specimen, for example, the heat-transfer coefficient varies by roughly a factor of 3 over a period of 300 sec during the heating process. This variation does not take place just during the initial period, where the rate of change of surface temperature is relatively high, but is roughly uniform over the entire heating process.

Moreover, the heat-transfer coefficient at any instant, under identical external conditions, is greater for the more massive specimens.

Direct interferometer measurement of the temperature gradient in the boundary layer [24] has confirmed that specimen size influences the heat-transfer coefficient.

This result contradicts the conclusions of the theoretical and experimental studies mentioned above.

As we have mentioned, for a more massive body, the time rate of change in the surface temperature is smaller than for a body with lower heat capacity (the heat capacity can be determined by both the dimensions and the thermal characteristics). All other conditions being equal, therefore, the probability that the temperature field will lag in the boundary layer is greater for a body with lower heat capacity. In this case, we can also expect a greater variation in the heat-transfer coefficient as compared with its quasistationary value.

During a heating (cooling) process, as the rate of change in the temperature diminishes, a time interval must occur for which the heating process will begin to be quasistationary. As reported in [28, 36, 37, 29], for cooling in air, either a quasistationary process sets in after 5-15 sec, or the process was quasistationary from the very beginning. For a specimen heated in water, in accordance with the conclusions of the theoretical studies, the unsteady process should last longer than for air. A quasistationary process will also inevitably occur in this case as well, however. For the present study, the unsteady process continued almost to the end of heating. Moreover, the values obtained for the change in the unsteady heat-transfer coefficient (by a factor of 3-6) are too high for such conditions, in our opinion.

Nonetheless, the fundamental notions as to unsteady heat transfer first formulated in [4] are valid and represent a point of departure for further research in this area.

In [44] it was assumed that there can in general be no relationship between the heat-transfer coefficient and the physical properties or dimensions of the specimens. In the opinion of the author of [44], the results obtained in [4] are accounted for by the fact that a one-dimensional treatment was used, while the temperature field in the specimen was essentially not one-dimensional. This fact is associated with the variation in the heat-transfer coefficient along the diameter of the end of the specimen.

It must be noted that special measures were employed to create a one-dimensional temperature field [4], while the report gave experimental data confirming that the one-dimensional condition was satisfied. It is also clear that the data obtained referred to the average-transfer coefficient for the end of the specimen. There have been numerous published studies in which it is the heat-transfer coefficients averaged over a surface that have been obtained, and this in no way interfered with the derivation of valid physical relationships. Thus the explanation of the results of [44] given in [4] seems unconvincing to us.

The work reported in [20-23, 32] represents one of the most careful experimental investigations of unsteady heat exchange under conditions resembling those in actual tube heat exchange (in reactor fuel elements, for example).

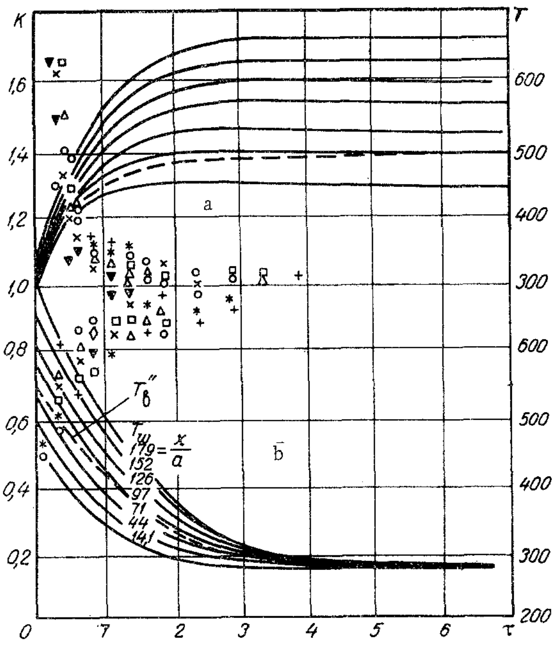


Fig. 4. Change in wall temperature $T_w$ (°K), parameter K for seven values of x/d, and outlet temperature $T_a''$ with time (tube, δ = 0.225 mm) [23]: a) load pickup ($Re_1 = 41.2 \cdot 10^4$; $Re_2 = 33.2 \cdot 10^4$); b) load shedding ($Re_1 = 15.6 \cdot 10^4$; $Re_2 = 20.4 \cdot 10^4$).

The experimental object was a stainless-steel tube, about 5 mm in diameter, with 0.3 mm wall thickness and lengths of about 1 m. Air was passed through the tube at a monitored flow rate. Unsteady heat exchange was set up by varying the heat released in the tube wall through a change in the current passed through it. In an experiment employing Chromel–Alumel thermocouples 0.1 mm in diameter and a type N-700 oscilloscope, the temperature of the outside tube surface was recorded at various distances from the inlet section. Heat insulation was used on the outside to reduce tube heat losses. The air temperature was also measured at the inlet and outlet of the experimental section, as was the current and the voltage drop across the tube, and the air flow rate and pressure.

The following quantities were calculated from the experimental data by means of the method discussed above: $Nu_n$, Re, Pr, $T_w/T_a$, $K_t$, and $K_G$, where

$$K_\tau = \frac{\partial T_w d^2}{\partial \tau (T_w - T_a) a_f}, \quad K_G = \frac{\partial G}{\partial \tau} \frac{d^2}{G\nu}.$$

The values of Nu were determined experimentally under steady-state conditions. They agree with the published data. The authors propose that $K_t$ and $K_G$ be taken as the fundamental parameters determining the ratio $K = Nu_n/Nu_{st}$, on the basis of an analysis of the equation system. In essence, $K_t$ is the heating (cooling) rate in dimensionless form; it represents the rate of change in the dimensionless surface temperature with time.

Figures 4 and 5 show results of experiments involving load pickup and shedding as a function of these parameters. The parameters were varied over the following ranges: $Re = (2\text{-}4) \cdot 10^4$, $T_w/T_a = 1.0\text{-}1.65$, $K_t = -30$ to 30. The rate of change in the wall temperature reached 200-360°C/sec.

The experiments demonstrated that under unsteady conditions the heat-transfer coefficient will differ from the steady-state values. With an increase in the rate of change of the surface temperature (or $K_t$), this difference increases. The ratio $K = Nu_n/Nu_{st}$ reached 1.6-1.8. The experiments confirmed that $Nu_n$ and $Nu_{st}$ depend in the same way on Re. Critical relationships were obtained that make it possible to determine the intensity of unsteady heat transfer for air flowing in a tube over the investigated range of parameters. The experimental results are in agreement with the process model. The relationships obtained make it possible to estimate the quasisteady and unsteady regions. For the case considered, K exceeds 1.05, provided the rate of change in the surface temperature exceeds 5-10°C/sec.

For the tube used in the experiments (diameter 5.5 mm, wall thickness 0.225 mm) the Bi number is estimated at 0.044. Thus we can employ the method considered above for determining the heat-transfer coefficient at low Bi, which is simpler than the method employed in the given study. It would be useful to compare the results obtained by the two methods so as to estimate the accuracy of the results. For small Bi, the temperature drops across the wall thickness are also small.

In the experiments, it took 1.2-20 sec to establish the new process, with the time varying roughly in proportion to the tube thickness (wall thicknesses of 0.225 mm and 0.32 mm were used). Thus the settling time and the rate of change in the wall temperature were affected both by the rearrangement of the temperature profile in the boundary layer and the tube heat capacity. The process studied represents a combination of cooling (heating) of the object and unsteady heat exchange. No special investigation was made of the relationship between the heat-exchange rate and the wall thickness, but in principle this relationship participates in the ultimate results, since the rate of change of the surface temperature depends on the wall thickness. From the engineering viewpoint, it would be convenient to use K as a function of the thickness and

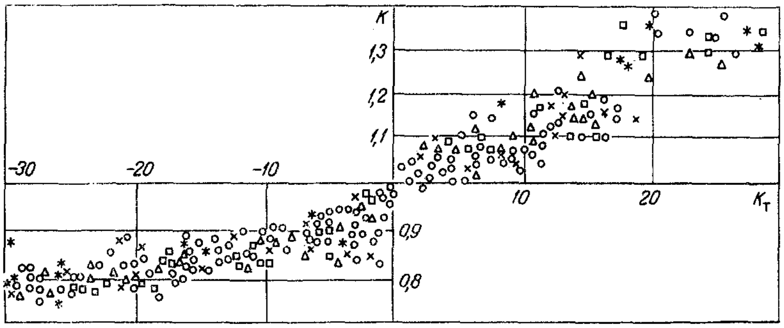


Fig. 5. Dependence of K on $K_t$ for step variation in heat release in tube wall [23].

properties of the wall material, the fluid temperature, and the heat liberated in the wall, and not as a function of the rate of change of temperature at the surface, referred to the difference $T_w - T_a$, since we do not know the latter quantities in advance.

There was no mention in the study of how the time derivatives of the temperature, which occur in $K_t$, were determined from the experimentally measured temperature–time relationships. Certain error can occur when a graphical method is used. Moreover, errors can occur when the tube surface temperature is measured by means of thermocouples with junction diameters of about 0.2 mm when the tube thickness is 0.225-0.32 mm. It is not clear whether or not the entire junction was placed within the wall, or merely touched a portion of the tube surface. In either case, errors can occur. For the tube temperatures (up to 500-700°C), there can be substantial losses to the insulation, and substantial losses to the ambient.

We can draw several general conclusions from the analysis of the theoretical and experimental results.

1. The two heat-transfer cases considered (steady-state and unsteady) differ substantially. Calculations for unsteady heat transfer by the methods developed for steady-state cases give false results when the instantaneous values of the parameters are used.

2. The unsteady heat-transfer coefficient is a function of the time. Under identical conditions and, in particular, for identical temperature differences $(t_w - t_f)$, the rate of unsteady heat transfer differs from from the rate of steady heat transfer.

3. For the two cases discussed, the ratio of these rates depends on the fluid flow parameters (Re, Pr), the properties (physical parameters and thickness) of the wall, and the time (or on the time rate of change in the surface temperature).

4. The features of unsteady heat exchange are independent of the flow regime, the shape of the body, the nature of the flow past the body (external flow, tube flow); there will be a specific relationship for each particular case.

Our discussion is not valid for certain extraordinary conditions, but will hold for many problems encountered in practice. This is true whether we have a step variation in the heat-transfer parameters, or whether they vary monotonically. The degree to which the process deviates from the steady state depends on an entire complex of parameters, and in particular on $K_t = (d^2/a(t_w - t_{fl}))(\partial t/\partial \tau)$. Thus, for example, when the surface temperature varies monotonically, the amount by which the heat-transfer coefficient deviates from the steady-state value for large Pr may be greater than for a step change in the temperature by the same amount, but with a smaller value of Pr.

5. None of what we have said should be interpreted as excluding quasisteady processes of unsteady heat transfer when the relationships of parameters are suitable. Certain studies have established values of parameters that delimit the regions of unsteady and quasisteady heat transfer.

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